Accurate Reduced-Order Models for Heterogeneous Coherent Generators

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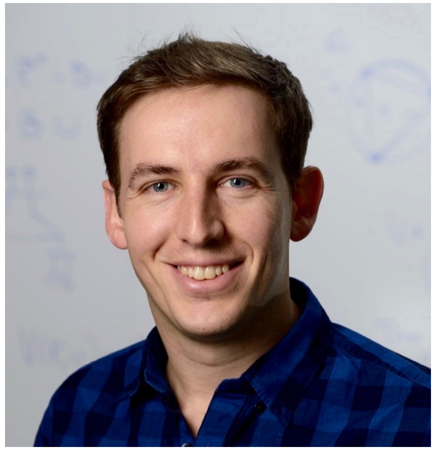
Acknowledgements



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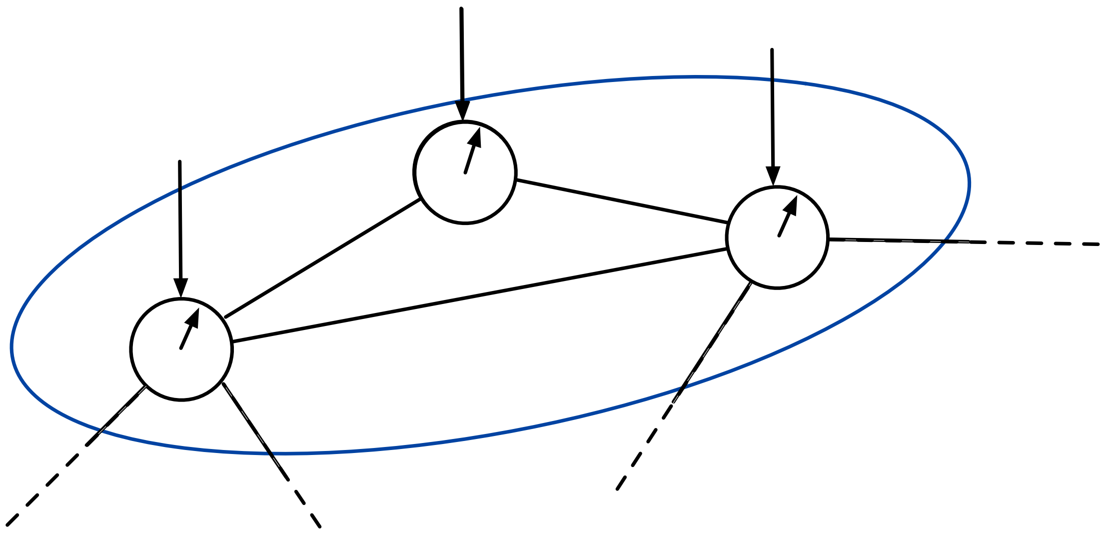
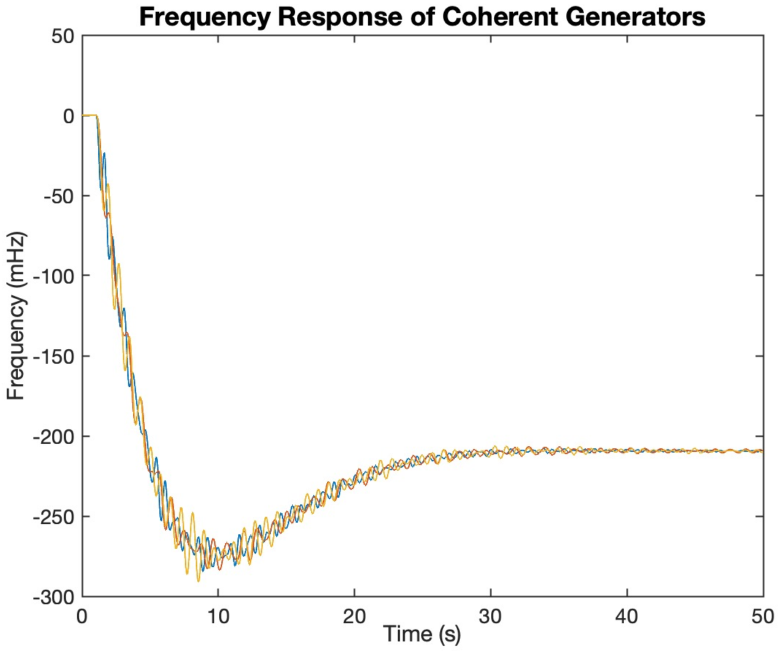
Coherent Group of Synchronous Generators

A group of generators are **coherent** if they have similar frequency response under disturbances.



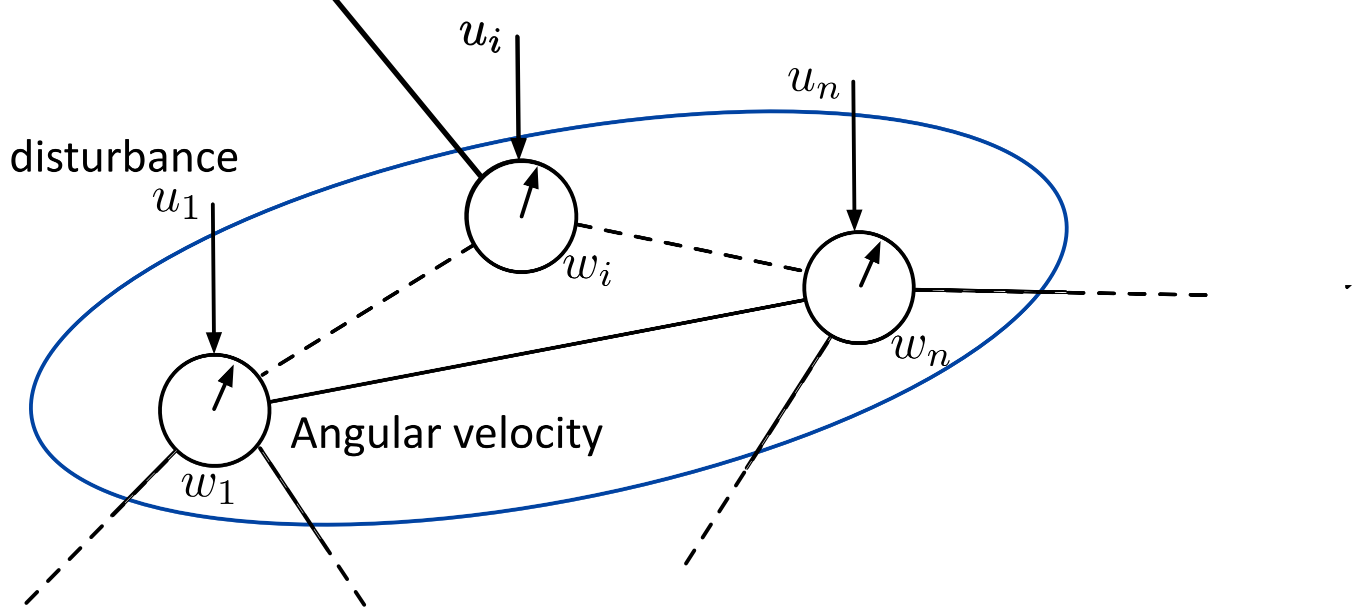


When modeling the frequency response of power networks, each group of coherent generators are aggregated into a single effective machine.

Aggregation of Coherent Generators

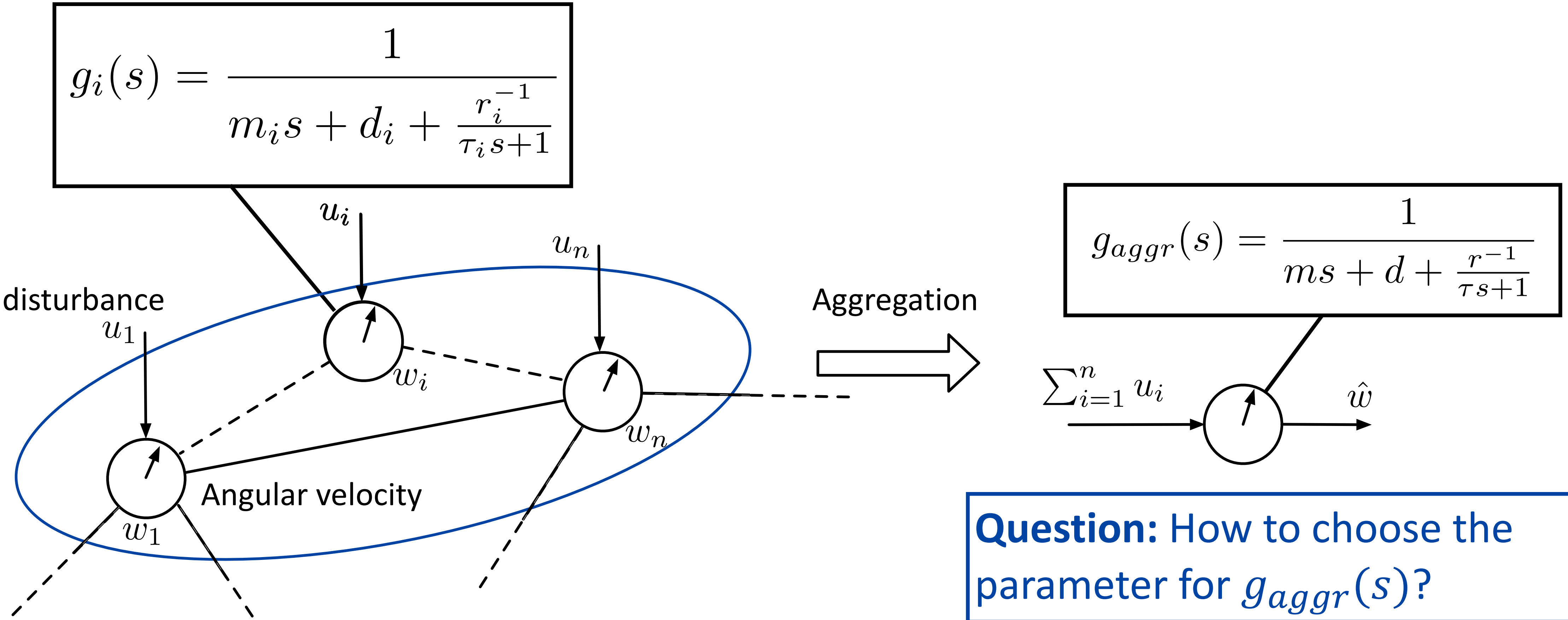
$$\boxed{g_i(s) = \frac{1}{m_i s + d_i + \frac{r_i^{-1}}{\tau_i s + 1}}}$$

m_i: inertia
d_i: damping coefficient
r_i^{-1}: droop coefficient
τ_i: turbine time constant



coherent group of n generators

Aggregation of Coherent Generators

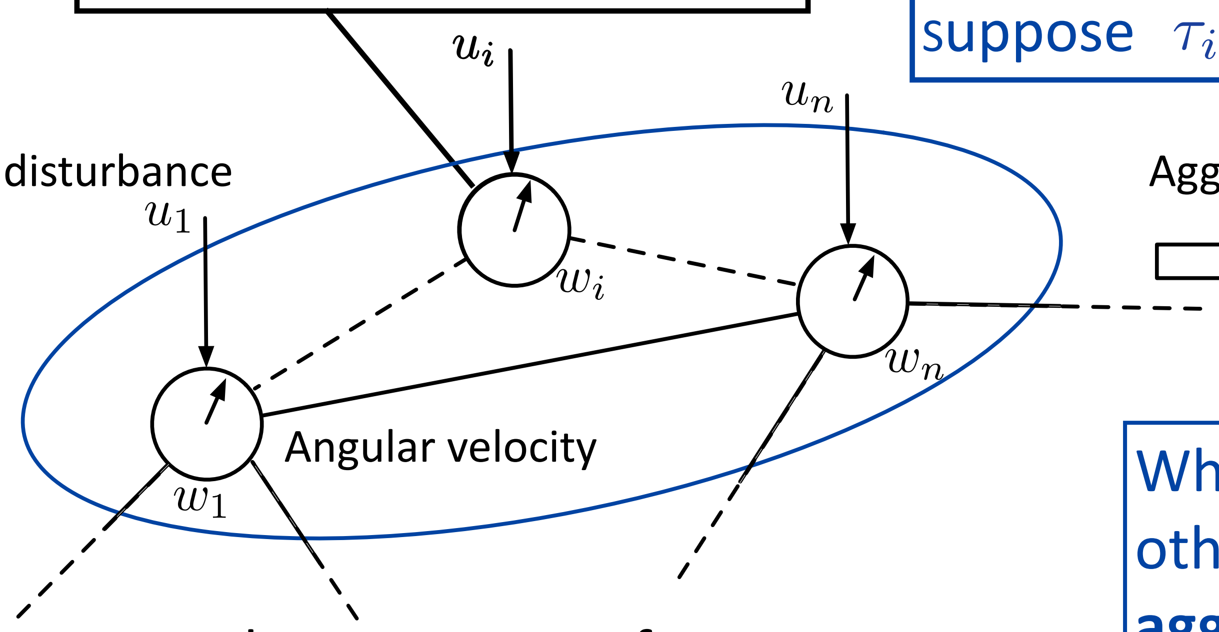


coherent group of n generators

Question: How to choose the parameter for $g_{aggr}(s)$?

Aggregation for Homogeneous $\tau_i = \tau$

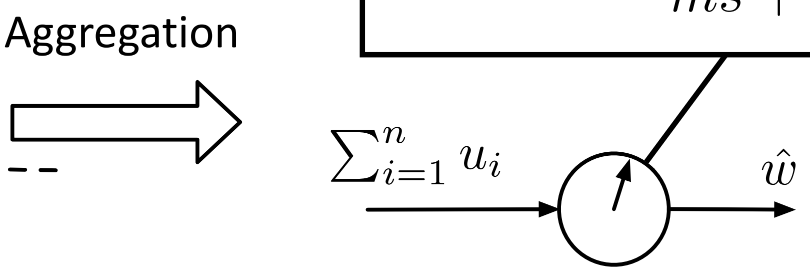
$$g_i(s) = \frac{1}{m_i s + d_i + \frac{r_i^{-1}}{\tau_i s + 1}}$$



suppose $\tau_i = \tau$

$$\hat{m} = \sum_{i=1}^{n} m_i, \quad \hat{d} = \sum_{i=1}^{n} d_i, \quad \hat{r}^{-1} = \sum_{i=1}^{n} r_i^{-1}$$

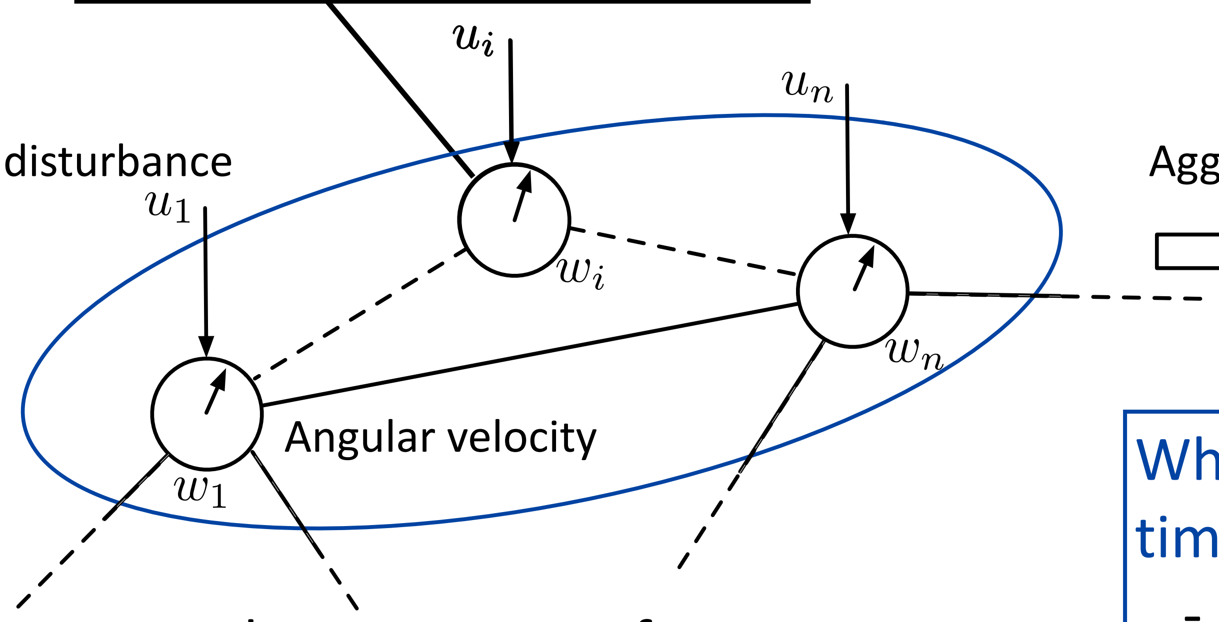
$$g_{aggr}(s) = \frac{1}{\hat{m}s + \hat{d} + \frac{\hat{r}^{-1}}{\tau s + 1}}$$



When time constants are **homogeneous**, all other parameters are chosen to be the **aggregated** value

Aggregation for heterogeneous τ_is

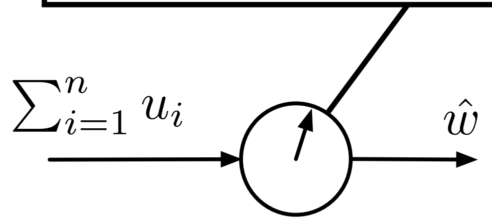
$$g_i(s) = \frac{1}{m_i s + d_i + \frac{r_i^{-1}}{\tau_i s + 1}}$$



$$\hat{m} = \sum_{i=1}^{n} m_i, \quad \hat{d} = \sum_{i=1}^{n} d_i, \quad \hat{r}^{-1} = \sum_{i=1}^{n} r_i^{-1}$$

Aggregation

$$g_{aggr}(s) = \frac{1}{\hat{m}s + \hat{d} + \frac{\hat{r}^{-1}}{\tau s + 1}}$$



When time constants are **heterogenous,** time constant τ is chosen by:

- Optimization: Germond'78, Guggilam'18
- Weighted harmonic mean: Ourari'06

A. J. Germond, et al., “Dynamic aggregation of generating unit models,” IEEE Transactions on Power Apparatus and Systems, vol. PAS-97, no. 4, pp.1060–1069, July 1978.
S. S. Guggilam, et al., “Optimizing DER participation in inertial and primary-frequency response,” IEEE Trans. Power Syst., vol. 33, no. 5, pp. 5194–5205, Sep. 2018.
M. L. Ourari, et al., "Dynamic equivalent modeling of large power systems using structure preservation technique," in *IEEE Transactions on Power Systems*, vol. 21, no. 3, pp. 1284-1295, Aug. 2006.

Aggregation for heterogeneous τ_is

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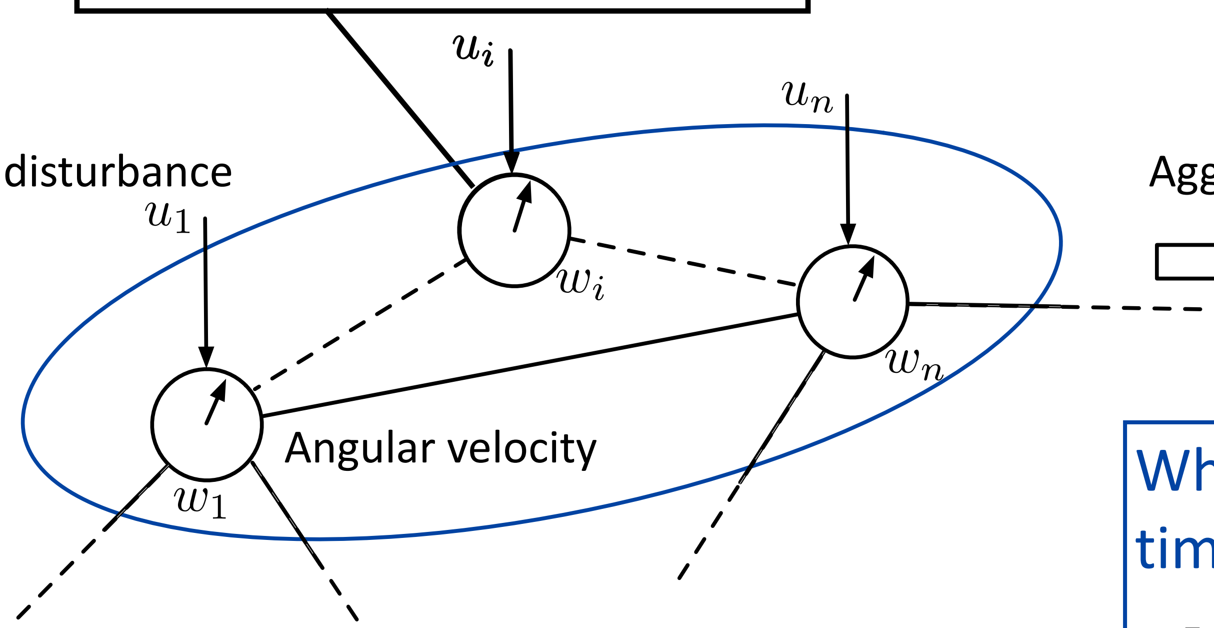

coherent group of n generators

Aggregation

$$\hat{m} = \sum_{i=1}^{n} m_i, \quad \hat{d} = \sum_{i=1}^{n} d_i, \quad \hat{r}^{-1} = \sum_{i=1}^{n} r_i^{-1}$$

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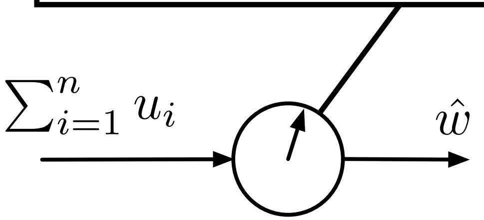

When time constants are **heterogenous,** time constant τ is chosen by:

- Optimization: Germond'78, Guggilam'18
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Drawbacks:

- Approximation model is restricted to 2nd order
- The only "decision variable" is the time constant

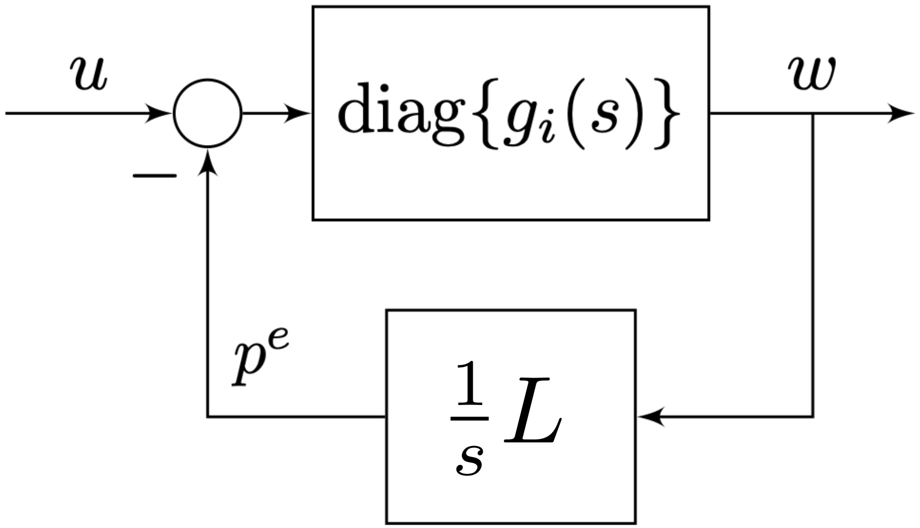
Inaccurate Approximation

This Talk

- Characterization of **coherent dynamics** [CDC'19]
- Reduced order model for coherent generators [ACC21/L-CSS]
- Numerical illustrations

Characterization of Coherent Dynamics

Block Diagram (Linearized around equilibrium):



$g_i(s), i = 1, \cdots, n$: Generator dynamics,

$L = [L_{ij}]$,

$L_{ij} = \frac{\partial}{\partial \theta_j} \sum_{k=1}^{n} |V_i||V_k| b_{ik} \sin(\theta_i - \theta_k)\Big|_{\theta=\theta_0}$

L symmetric, $0 = \lambda_1(L) \leq \lambda_2(L) \leq \cdots \leq \lambda_n(L)$

- When the entire network is considered coherent, what is the coherent dynamics?

$$\boxed{\hat{g}(s) = \left(\sum_{i=1}^{n} g_i^{-1}(s)\right)^{-1}}$$

- When does the network have strong coherence?

Algebraic connectivity $\lambda_2(L)$ is a measure of coherence

Coherence/Dynamics Concentration of Coupled Generators

Theorem. *Assume all* $g_i(s)$ *are positive real. Let the transfer matrix from* u *to* w *be* $T(s)$*, then for any* $\eta_0 > 0$*:*

$$\lim_{\lambda_2(L)\to+\infty} \sup_{\eta\in[-\eta_0,\eta_0]} \left\|T(j\eta) - \hat{g}(j\eta)\mathbb{1}\mathbb{1}^T\right\| = 0, \quad j = \sqrt{-1}.$$

$$\boxed{\hat{g}(s) = \left(\sum_{i=1}^{n} g_i^{-1}(s)\right)^{-1}}$$

- Uniform convergence of $T(s)$ on low frequency band
- The limiting dynamics $\hat{g}(s)\mathbb{1}\mathbb{1}^T$ is coherent
- Algebraic connectivity of L is an indicator of **network coherence**
- $\hat{g}(s)$ accurately represents the aggregate dynamics in the **asymptotical sense**
- Extension of result (CDC '19) on **coherence in networked dynamical systems**. A more comprehensive analysis is available (arXiv 2101.00981, under review)

H. Min and E. Mallada, "Dynamics Concentration of Tightly-Connected Large-Scale Networks," in 58th IEEE Conference on Decision and Control (CDC), pp. 758-763, 2019.
H. Min and E. Mallada, "Coherence in tightly-connected networks,"arXiv preprint arXiv:2101.00981, 2021.

Challenges on Aggregating Coherent Generators

For generator dynamics given by a swing model with turbine control:

$$g_i(s) = \frac{1}{m_i s + d_i + \frac{r_i^{-1}}{\tau_i s + 1}}$$

The aggregate dynamics:

$$\hat{g}(s) = \frac{1}{\hat{m}s + \hat{d} + \boxed{\sum_{i=1}^{n} \frac{r_i^{-1}}{\tau_i s + 1}}}$$

high-order if τ_i are heterogeneous

Need to find a low-order approximation of $\hat{g}(s)$

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Aggregation Model by Balanced Truncation

Two approaches for k-th order reduction model of $\hat{g}(s)$:

High-order Coherent Dynamics

$$\hat{g}(s) = \frac{1}{\hat{m}s + \hat{d} + \sum_{i=1}^{n} \frac{r_i^{-1}}{\tau_i s + 1}}$$

Problem:
DC gain of the reduced system
$\neq$DC gain of the original system

- **BTk-tb:** (k-1)-th order balanced truncation on high-order turbine dynamics

$$\tilde{g}_k^{tb}(s) = \frac{1}{\hat{m}s + \hat{d} + \boxed{\tilde{g}_{t,k-1}(s)}}$$

(k-1)-th reduction model on $\sum_{i=1}^{n} \frac{r_i^{-1}}{\tau_i s + 1}$

- **BTk-cl:**k-th order balanced truncation on closed-loop dynamics

k-th reduction model on $\hat{g}(s) = \left(\sum_i g_i^{-1}\right)^{-1}$

Aggregation Model by Frequency Weighted Balanced Truncation

Two approaches for k-th order reduction model of $\hat{g}(s)$:

High-order Coherent Dynamics

$$\hat{g}(s) = \frac{1}{\hat{m}s + \hat{d} + \sum_{i=1}^{n} \frac{r_i^{-1}}{\tau_i s+1}}$$

Problem:
DC gain of the reduced system $\neq$DC gain of the original system

Solution:
Use Frequency Weighted Balanced Truncation and put higher weights on low-frequency range

- **BTk-tb:** (k-1)-th order frequency weighted balanced truncation on high-order turbine dynamics

$$\tilde{g}_k^{tb}(s) = \frac{1}{\hat{m}s + \hat{d} + \boxed{\tilde{g}_{t,k-1}(s)}}$$

(k-1)-th reduction model on $\sum_{i=1}^{n} \frac{r_i^{-1}}{\tau_i s + 1}$

- **BTk-cl:** k-th order frequency weighted balanced truncation on closed-loop dynamics

k-th reduction model on $\hat{g}(s) = \left(\sum_i g_i^{-1}\right)^{-1}$

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Test Case Data

Synthetic test dataset, 5 generators in a coherent group

Aggregated inertia and damping:

$$\hat{m} = 0.0683, \quad \hat{d} = 0.0107$$

High-order Coherent Dynamics

$$\hat{g}(s) = \frac{1}{\hat{m}s + \hat{d} + \sum_{i=1}^{n} \frac{r_i^{-1}}{\tau_i s + 1}}$$

Turbine parameters:

Parameter \ Index	1	2	3	4	5
droop coeff. r_i^{-1}	0.0218	0.0256	0.0236	0.0255	0.0192
time constant τ_i	9.08	5.26	2.29	7.97	3.24

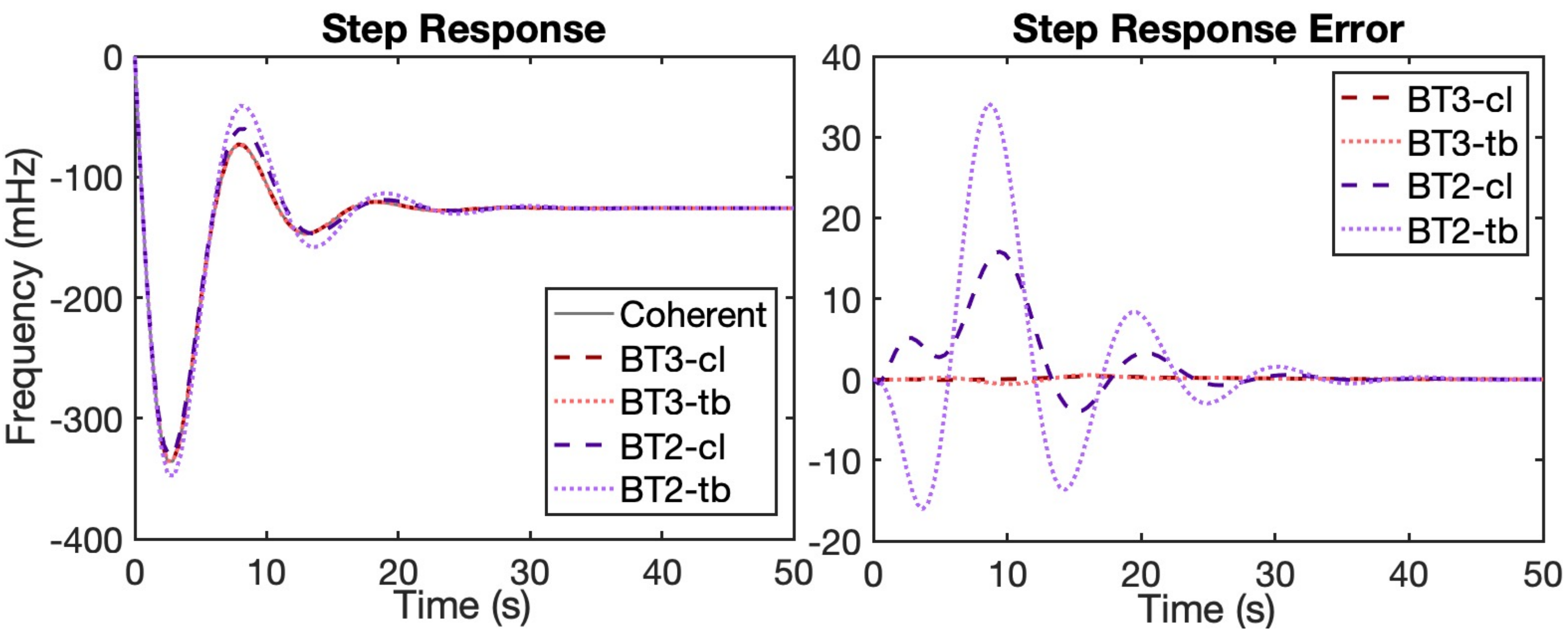
The time constants have wide spreading
It is hard for conventional approach to find a proper τ that accounts for both **fast** (3,5) and **slow** (1,2,4) turbines.

Comparison of Balance Truncation Models

We compare the following 4 reduced order models:

- BT on **turbines** with weight $W_{tb}(s) = \frac{s+3\cdot 10^{-2}}{s+10^{-4}}$
  - 2nd order (**BT2-tb**)
  - 3rd order (**BT3-tb**)

- BT on **closed-loop** with weight $W_{cl}(s) = \frac{s+8\cdot 10^{-2}}{s+10^{-4}}$
  - 2nd order (**BT2-cl**)
  - 3rd order (**BT3-cl**)



- 3rd order models are almost accurate
- balanced truncation on closed-loop is better than on turbine dynamics, given the same order.

Interpretation of 3rd Order Model – Reduction on Turbine

BT3-tb gives 2nd order reduction model on turbine dynamics:

$$\tilde{g}_{t,2}(s) = \frac{0.02653s + 0.005291}{s^2 + 0.4866s + 0.04572}$$

Partial fraction expansion gives:

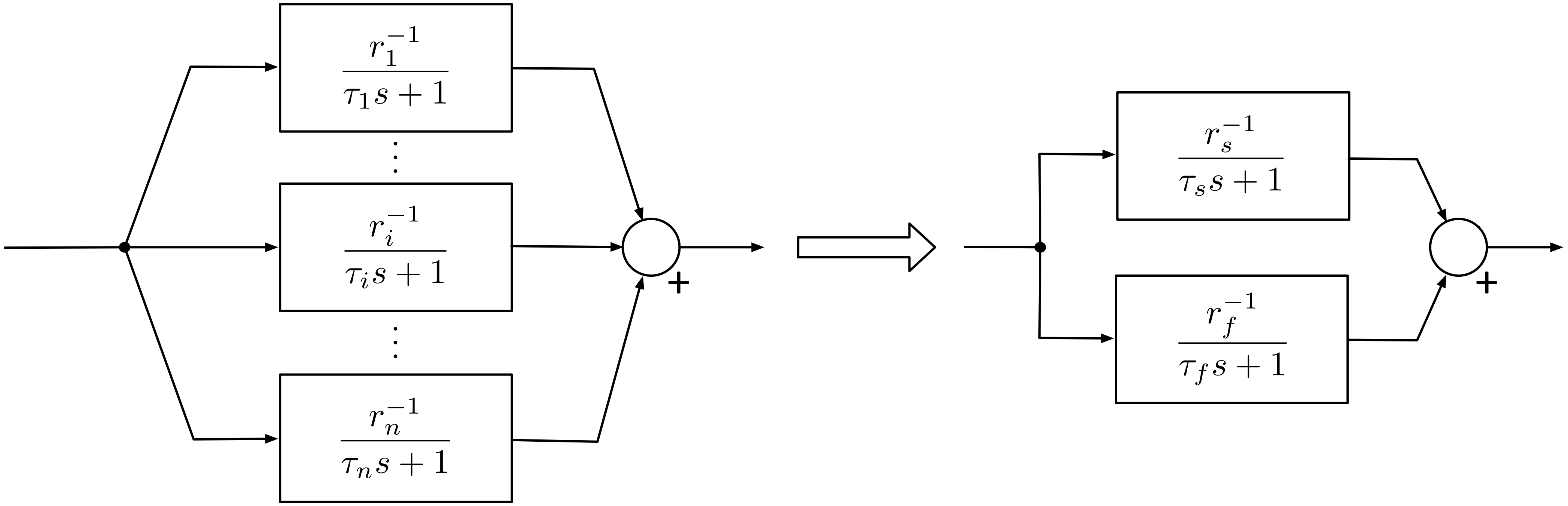
$$\tilde{g}_{t,2}(s) = \boxed{\frac{0.0509}{2.7827s+1}} + \boxed{\frac{0.0649}{7.8601s+1}}$$

Can be viewed as two first order turbines in parallel

Accounts for both slow and fast turbines

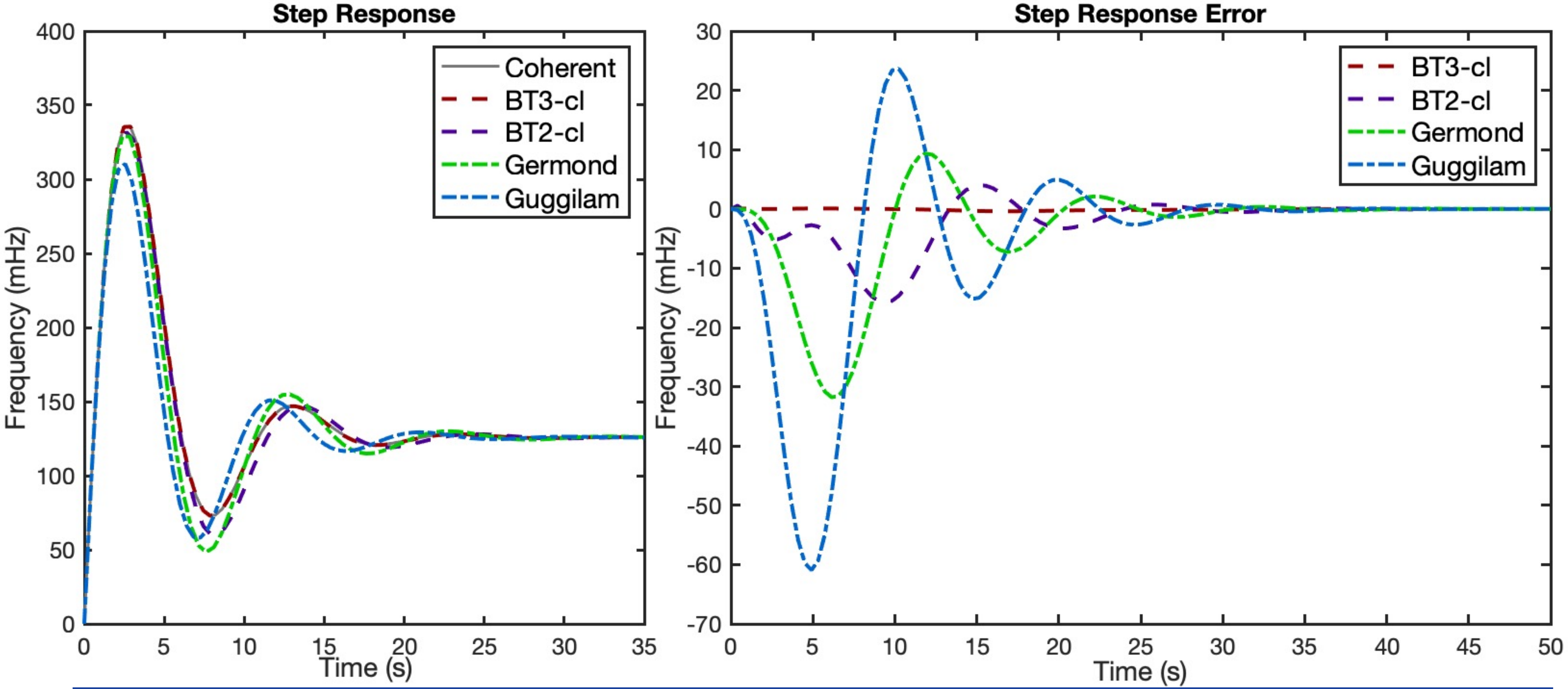
Parameter \ Index	1	2	3	4	5
droop coeff. r_i^{-1}	0.0218	0.0256	0.0236	0.0255	0.0192
time constant τ_i	9.08	5.26	2.29	7.97	3.24

Interpretation of 3rd Order Model – Reduction on Turbine



- The high-order turbine dynamics can be **almost accurately** recovered by **two turbines** in parallel
- Similar results obtained for larger test cases

Comparison with (Some) Existing Methods



By essentially relaxing the restrictions on reduced order model:

- **increase the model order to 3rd order,**
- **reduction on closed-loop dynamics,**

our proposed models outperform models by conventional approach

Conclusions

- We provide **theoretical justification** of the aggregate dynamics of coherent generators obtained by conventional approach
- We show that **algebraic connectivity** of graph Laplacian of linearized power network is a direct **indicator of the generator coherence**
- We use **frequency weighted balanced truncation** to suggest possible improvements to obtain accurate reduced order model of aggregated dynamics of coherent generators:
  - increase model complexity (3^{rd} order/two turbines)
  - model reduction on closed-loop dynamics

Thanks!

Related Publications:

- H. Min, F. Paganini, and E. Mallada, “Accurate Reduced Order Models for Coherent Heterogeneous Generators,” IEEE Control Systems Letters (L-CSS), pp. 1741-1746, 2020.
- H. Min and E. Mallada, “Dynamics Concentration of Tightly-Connected Large-Scale Networks,” in 58th IEEE Conference on Decision and Control (CDC), pp. 758-763, 2019.
- H. Min and E. Mallada, “Coherence in tightly-connected networks,”arXiv preprint arXiv:2101.00981, 2021.

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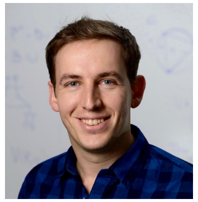

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