Spectral clustering and model reduction for weakly-connected coherent network systems

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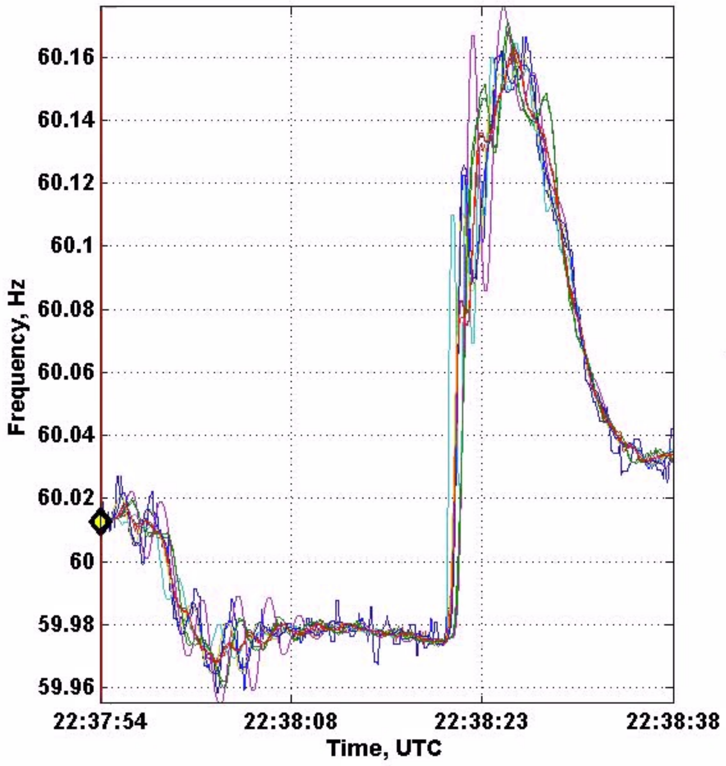
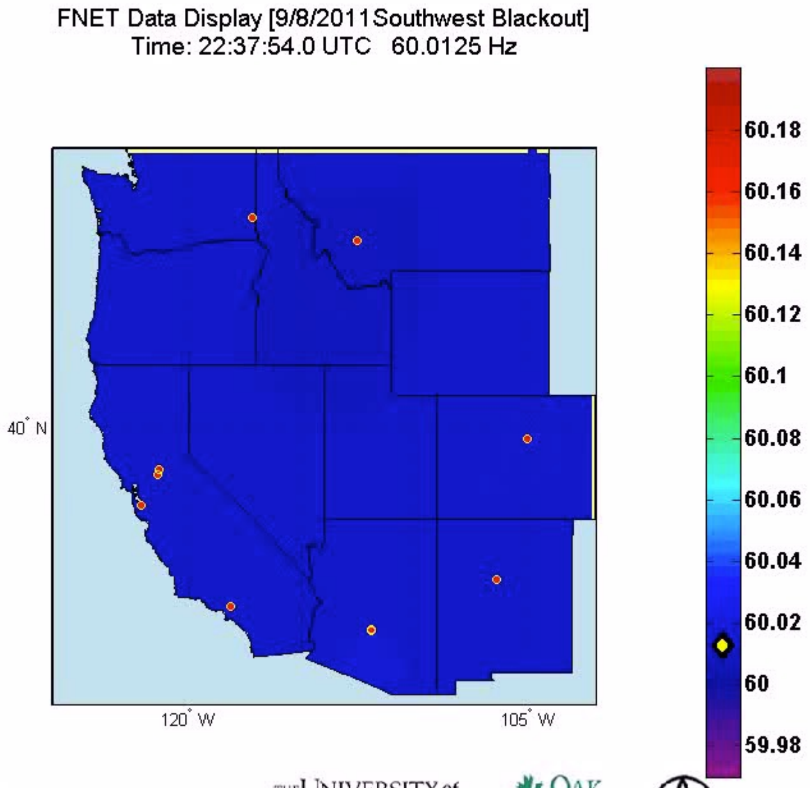
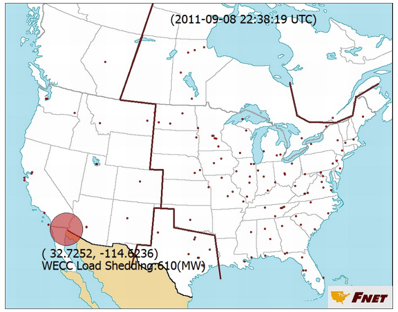
ACC 2023

Jun 1st

Motivation: Network Coherence



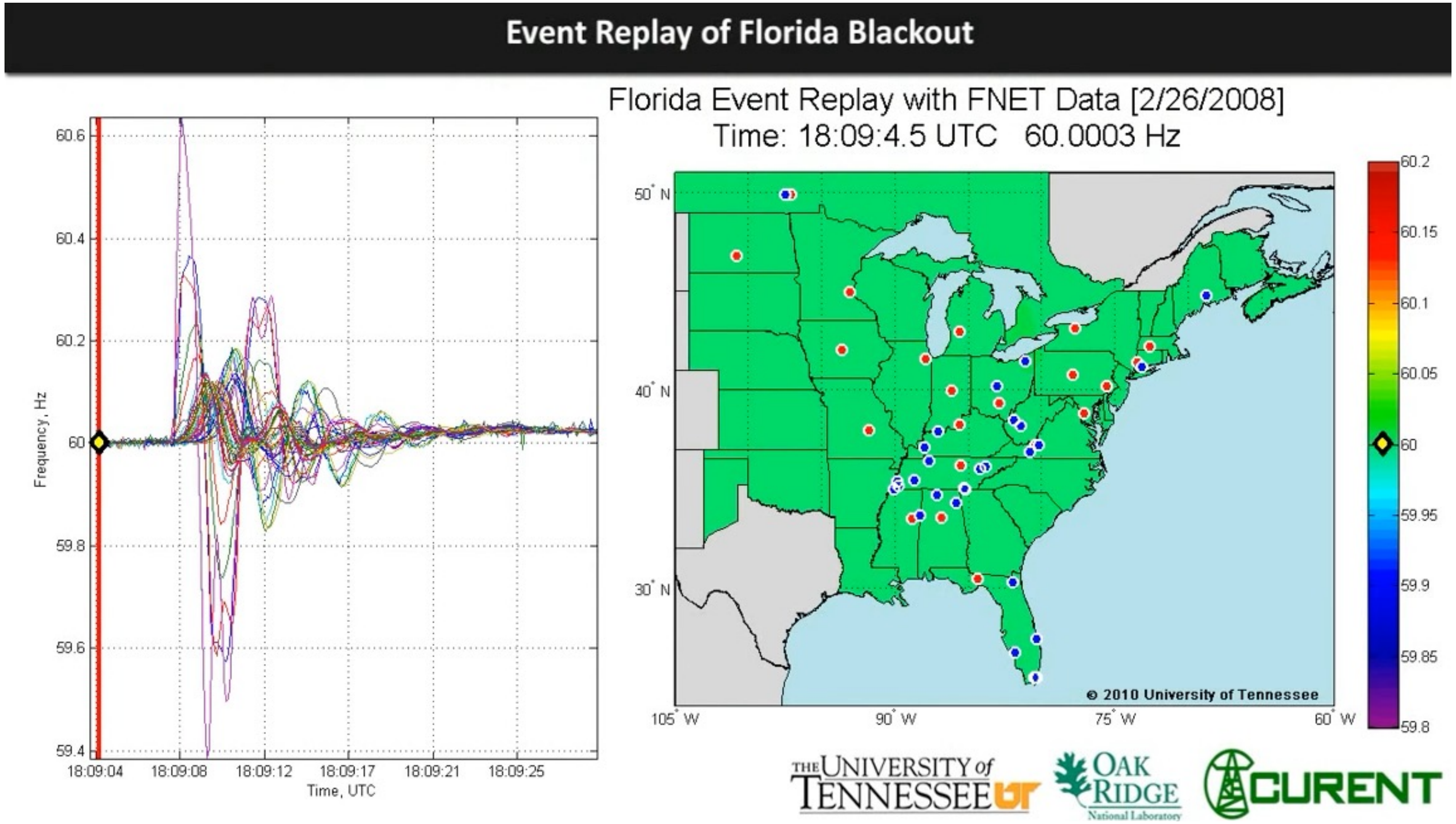




Key Questions:

- Why does coherence emerge?
- How to characterize the coherent response?

Motivation: Inter-area Oscillation



Key Questions:

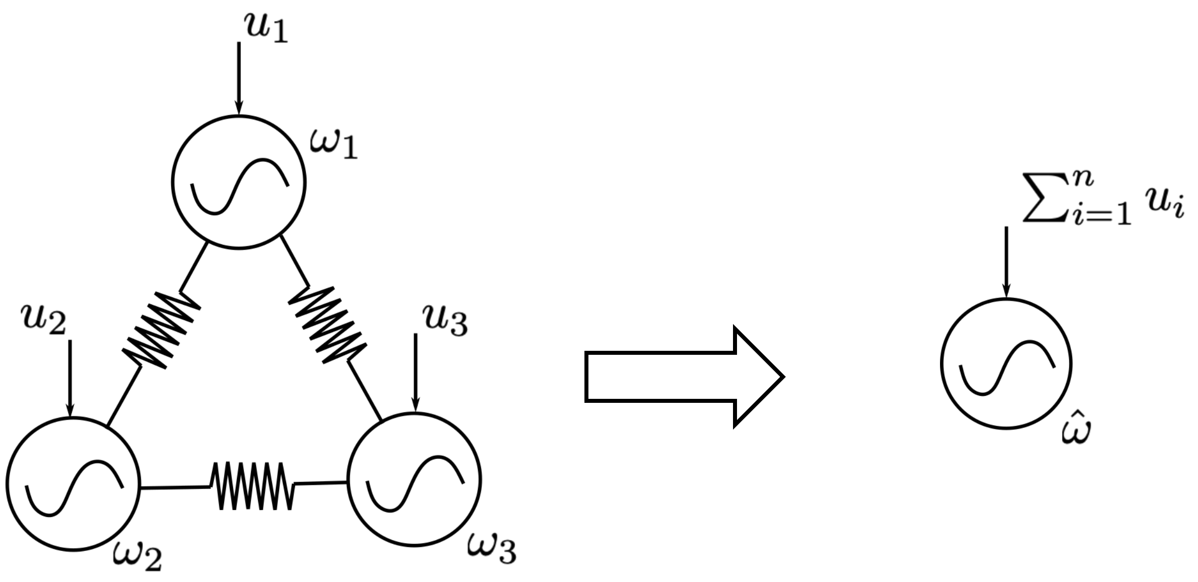
- How to identify coherent areas?
- Can we model the inter-area oscillations?

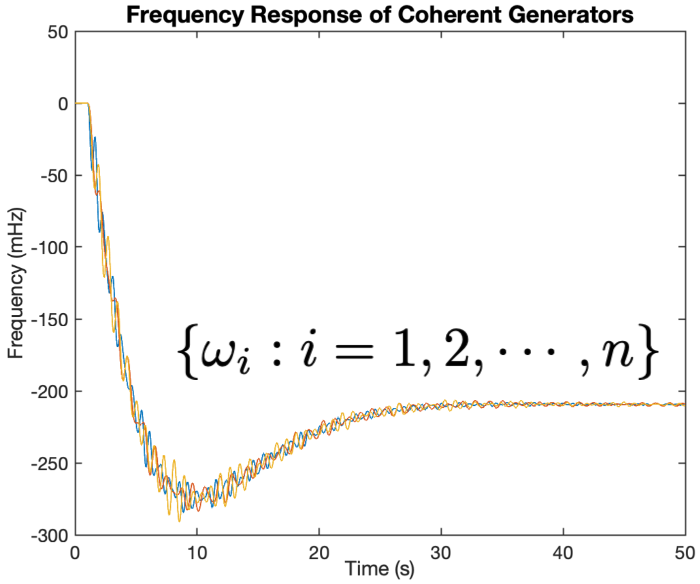
Coherence based aggregation

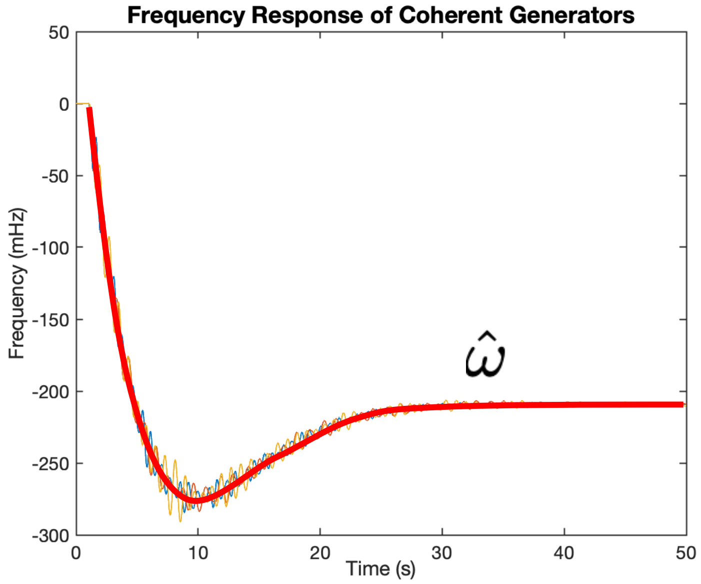
u_i: Power imbalance

ω_i: Frequency

- Coherent generators have similar frequency responses
- We should be able to **aggregate** coherent generators into a single one
- **Assess performance, control design** with the aggregate generator







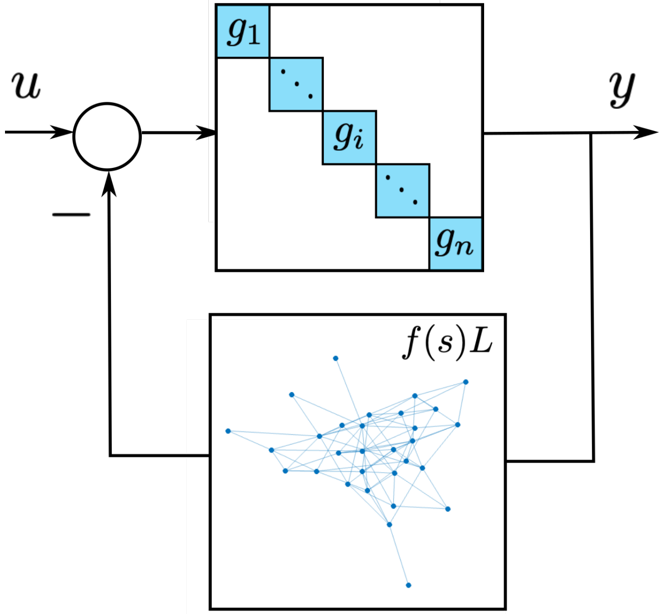
Prior Work

	Tool	Node Dynamics	Heterogeneous Node?	Input signal	Multi-area?
[Chow'82] [Romeres'13]	Singular Perturbation Analysis	First- or second-order LTI	Y	Any	Y
[Wu'83] [Sastry'81]	Markov Parameter	LTI	Y	Certain shape	Y
[Ramaswamy'95]	Invariant subspace	First- or second-order LTI	Y	Certain shape	Y
[Bamieh'12] [Paganini'20]	$\mathcal{H}_2$ Analysis	LTI	N	White noise	N
Our work	**Frequency domain Analysis**	LTI	Y	Any	Y

Outline

- Network coherence as low-rank structure in frequency domain
- Spectral clustering and model reduction for two weakly-connected areas
- Numerical verification
- Conclusion

Coherence as Low-rank Structure in Laplace Domain



Node dynamics: $g_i(s), i = 1, 2, \cdots, n$

Real symmetric graph Laplacian: L

$$L = V \Lambda V^T, \; V = [\mathbb{1}/\sqrt{n}, V_\perp]$$
$$\Lambda = \mathrm{diag}\{0, \lambda_2(L), \cdots, \lambda_n(L)\}$$

Coupling dynamics: $f(s)$

Low-rank structure in the closed-loop transfer matrix $T(s)$ from u to y:

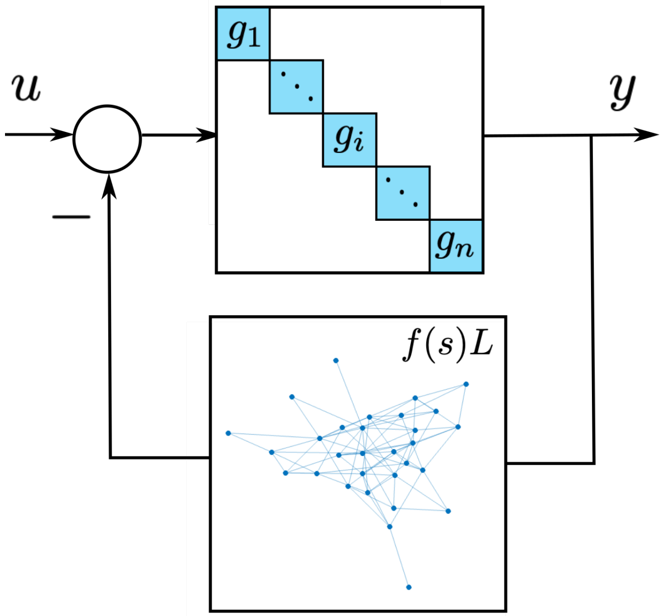
$$\|T(s_0) - \hat{g}(s_0)\mathbb{1}\mathbb{1}^\top\|_2 \sim \mathcal{O}\left(\frac{1}{\lambda_2(L)}\right), \qquad s_0 \in \mathbb{C}$$

$$\hat{g}(s) = \left(\sum_{i=1}^{n} g_i^{-1}(s)\right)^{-1}$$

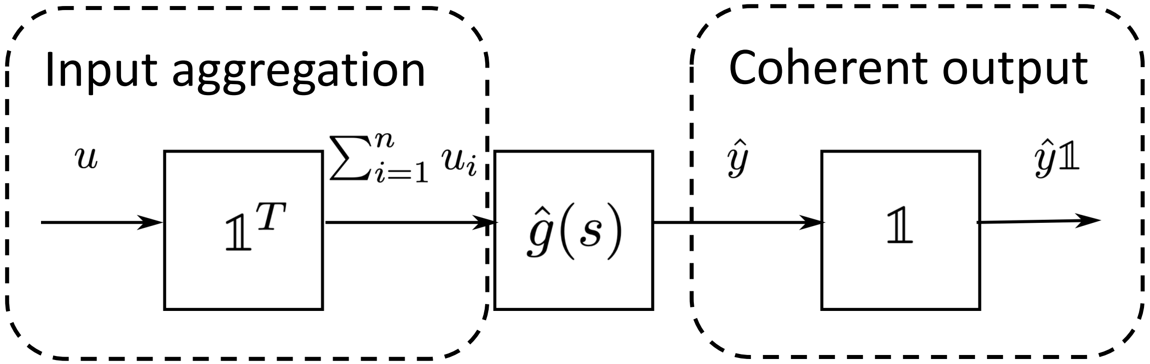
H. Min, R. Pates, and E. Mallada, "A Frequency Domain Analysis of Slow Coherency in Networked Systems," arXiv preprint arXiv:2302.08438, 2023

Coherence as Low-rank Structure of $T(s)$

$$\|T(s_0) - \hat{g}(s_0)\mathbb{1}\mathbb{1}^\top\|_2 \sim \mathcal{O}\left(\frac{1}{\lambda_2(L)}\right)$$



$T(s)$

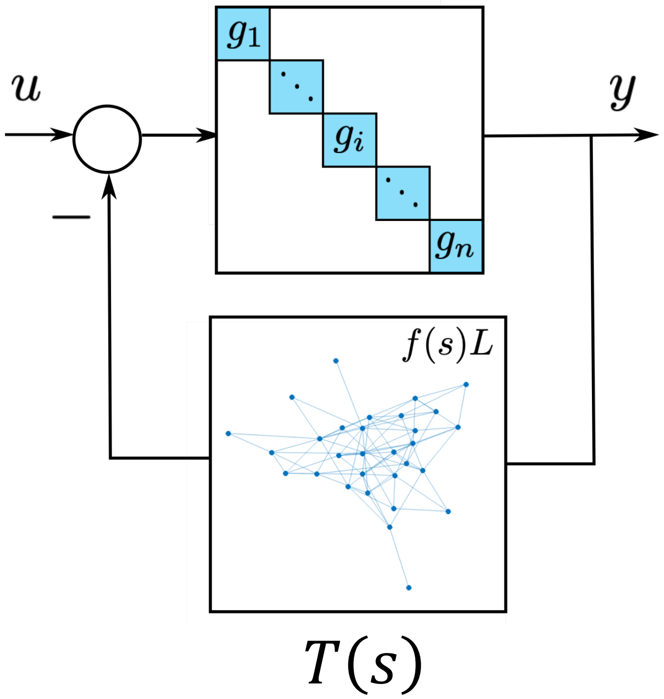


$\hat{g}(s)\mathbb{1}\mathbb{1}^\top$

Coherence emerges as connectivity increases: Error→ 0, if $\lambda_2(L) \to \infty$

Coherent response is characterized by $\hat{g}(s) = \left(\sum_{i=1}^n g_i^{-1}(s)\right)^{-1}$

Coherence as Low-rank Structure of $T(s)$: Example



Example: Linearized Power Networks

- Node dynamics: $g_i(s) = \frac{1}{m_i s + d_i}$
- Coupling dynamics: $f(s) = \frac{1}{s}$
- Coherent response: $\hat{g}(s) = \frac{1}{(\sum_i m_i)s + (\sum_i d_i)}$

Coherence emerges as connectivity increases: Error→ 0, if $\lambda_2(L) \to \infty$

Coherent response is characterized by $\hat{g}(s) = \left(\sum_{i=1}^{n} g_i^{-1}(s)\right)^{-1}$

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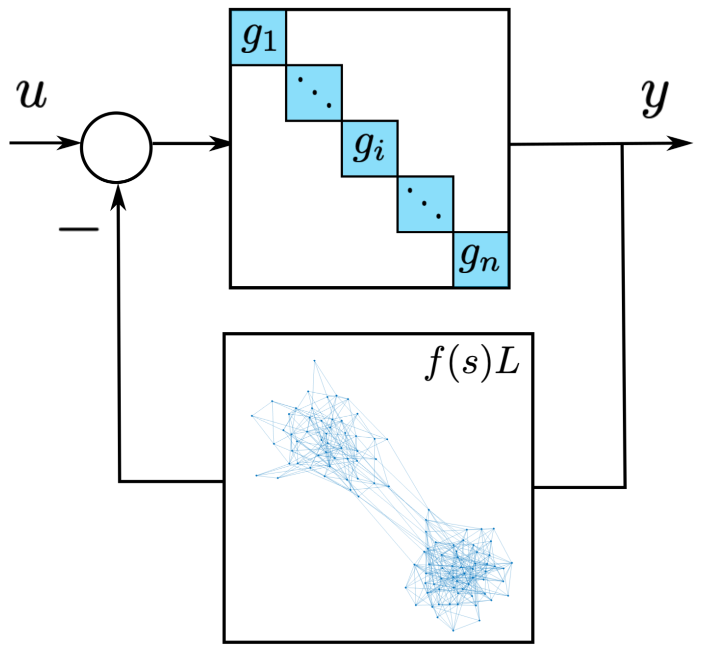
Low-rank structure in Weakly-Connected Coherent Networks



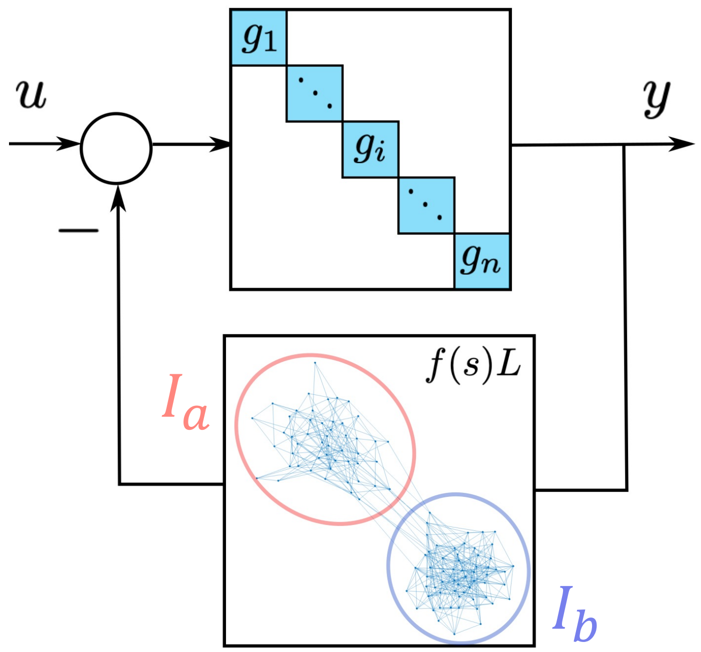
Two weakly-connected areas:

- Within each area, nodes are tightly-connected
- Weak/sparse connections between two areas

The graph Laplacian L:

- has small $\lambda_2(L)$, but a large $\lambda_3(L)$
- $v_2(L) \approx \begin{bmatrix} c_1\mathbb{1} \\ -c_2\mathbb{1} \end{bmatrix}$, for some positive c_1, c_2

Low-rank structure in Weakly-Connected Coherent Networks



We need to:

1. **Identify** two areas I_a, I_b.
   (Using some **spectral clustering** algorithm on L)

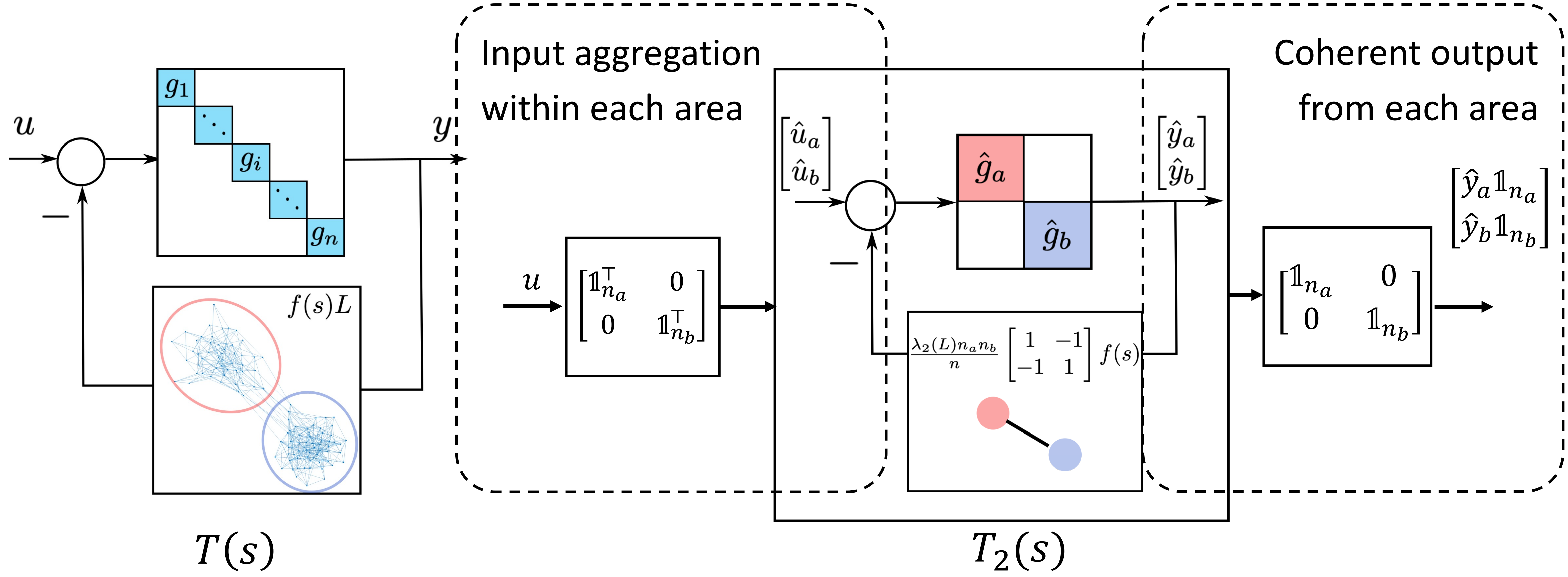
2. "**Aggregate**" the nodes within each area

$$\hat{g}_a(s) = \left(\sum_{i \in I_a} g_i^{-1}(s)\right)^{-1}, \hat{g}_b(s) = \left(\sum_{i \in I_b} g_i^{-1}(s)\right)^{-1}$$

3. Model the "**interaction**" between $\hat{g}_a(s)$ and $\hat{g}_b(s)$

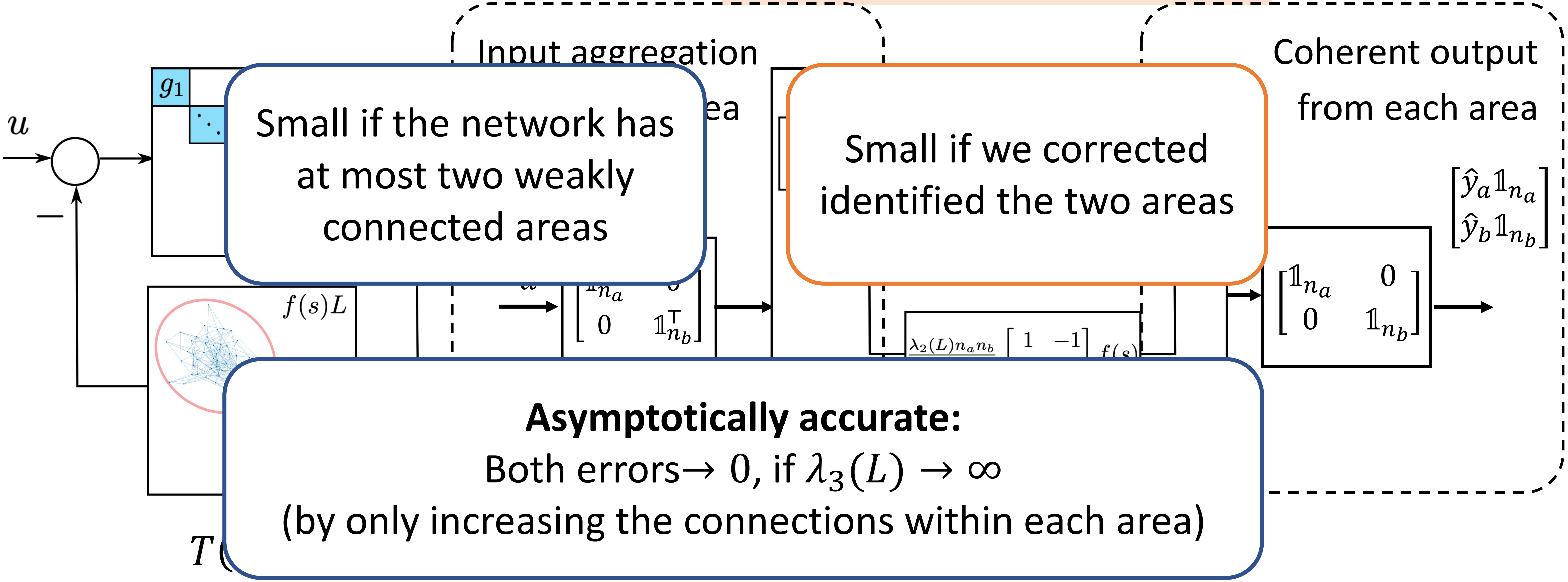
Low-rank structure in Weakly-Connected Coherent Networks

$$\|T(s_0) - T_2(s_0)\|_2 \sim \mathcal{O}\left(\frac{1}{\lambda_3(L)}\right) + \mathcal{O}\left(\left\| v_2(L) - \begin{bmatrix} \frac{\sqrt{n_b}}{\sqrt{n n_a}} \mathbb{1}_{n_a} \\ -\frac{\sqrt{n_a}}{\sqrt{n n_b}} \mathbb{1}_{n_b} \end{bmatrix} \right\|_2\right)$$



Low-rank structure in Weakly-Connected Coherent Networks

$$\|T(s_0) - T_2(s_0)\|_2 \sim \mathcal{O}\left(\frac{1}{\lambda_3(L)}\right) + \mathcal{O}\left(\left\| v_2(L) - \begin{bmatrix} \frac{\sqrt{n_b}}{\sqrt{n n_a}} \mathbb{1}_{n_a} \\ -\frac{\sqrt{n_a}}{\sqrt{n n_b}} \mathbb{1}_{n_b} \end{bmatrix} \right\|_2\right)$$

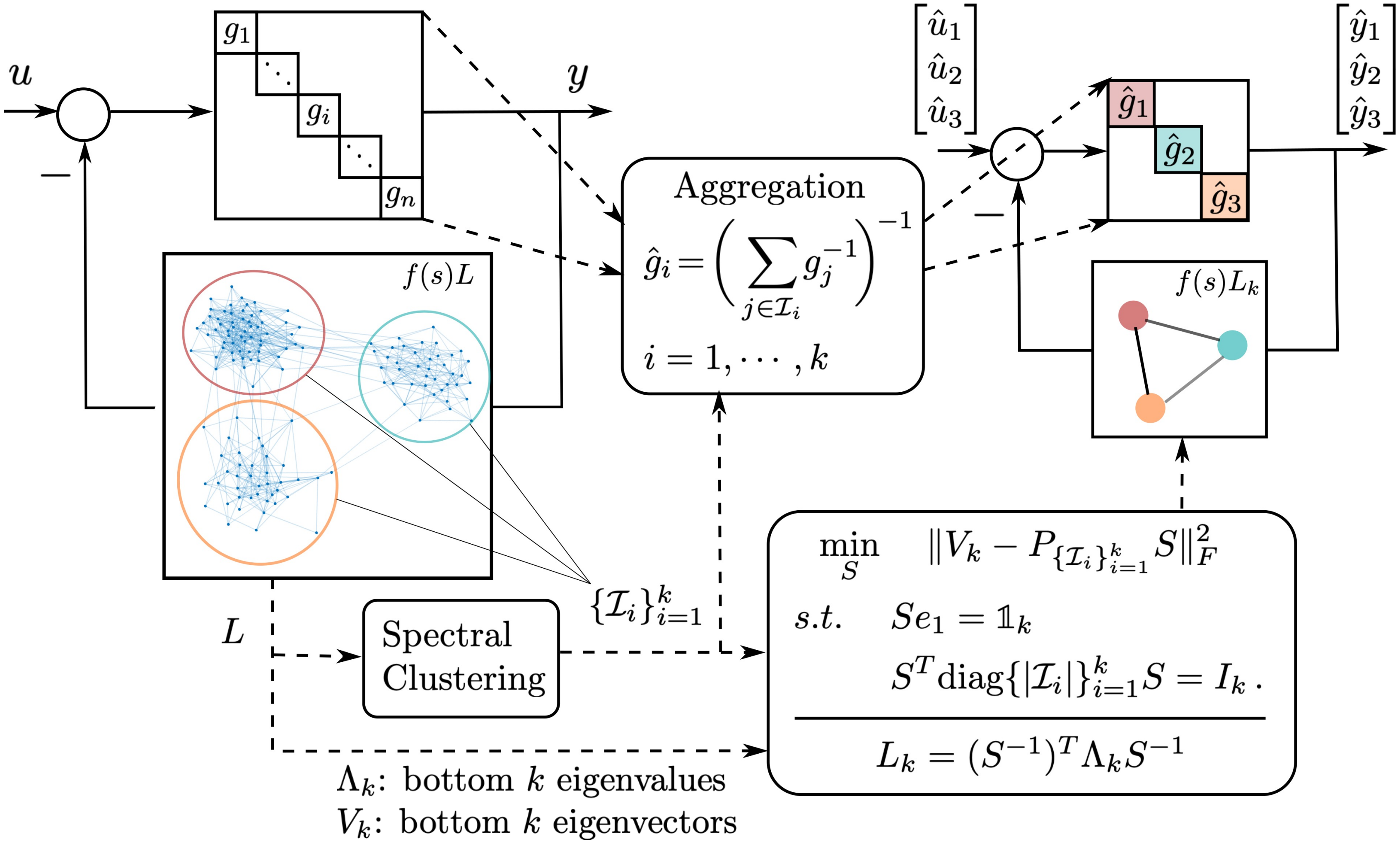


Small if the network has at most two weakly connected areas

Small if we corrected identified the two areas

Asymptotically accurate:
Both errors→ 0, if $\lambda_3(L) \to \infty$
(by only increasing the connections within each area)

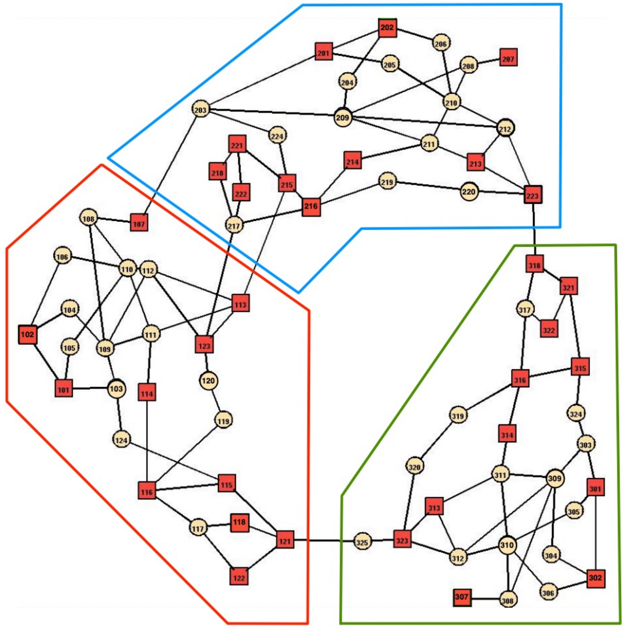
More Clusters?

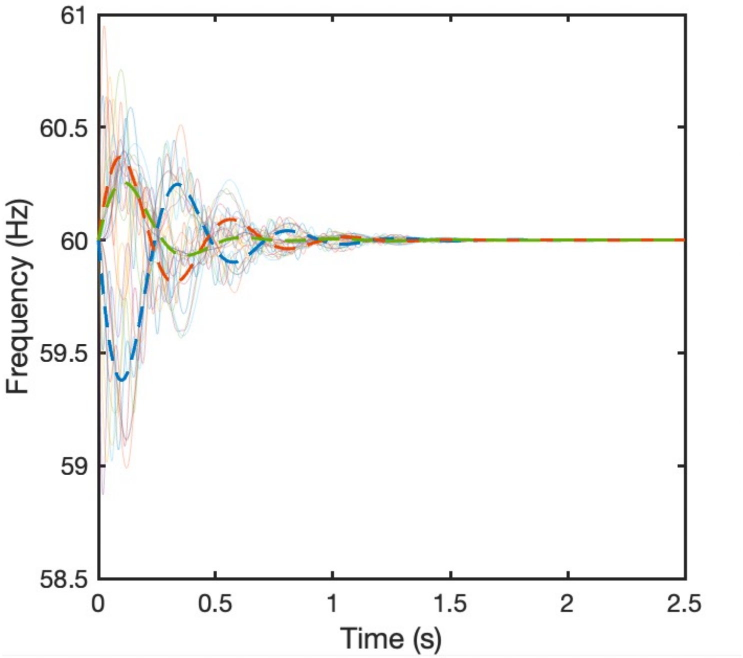


- **Spectral clustering** on graph Laplacian identifies coherent groups
- **Spectral embedding refinement** finds the interconnection
- **Structure-preserving** model reduction

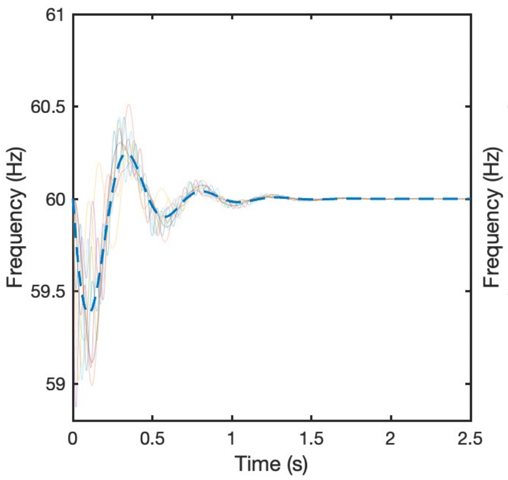
H. Min and E. Mallada, "Learning Coherent Clusters in Weakly-Connected Network Systems," arXiv preprint arXiv:2211.15301,

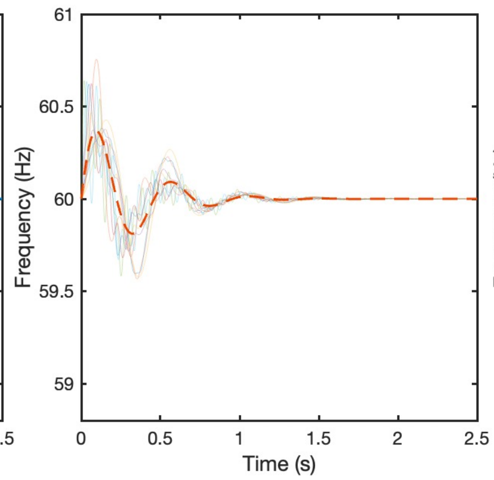
Numerical validation – RTS 96 test case

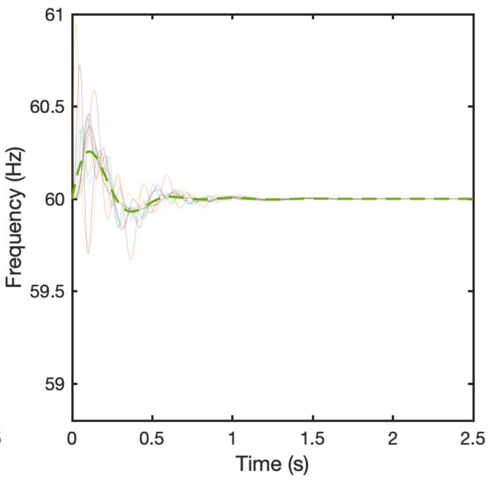




- The IEEE reliability test system: 1996
- 3 areas, 33 generators in total
- Different rotor angles across each area at initialization
- Solid lines: actual frequency response
  Dashed lines: reduced model







Conclusions

- Network coherence is related to a low-rank structure of the network transfer matrix
- We can build a reduced model based on such low-rank structure
- Spectral clustering techniques are used for identifying coherent areas