Learning Coherent Clusters in Weakly-Connected Network Systems

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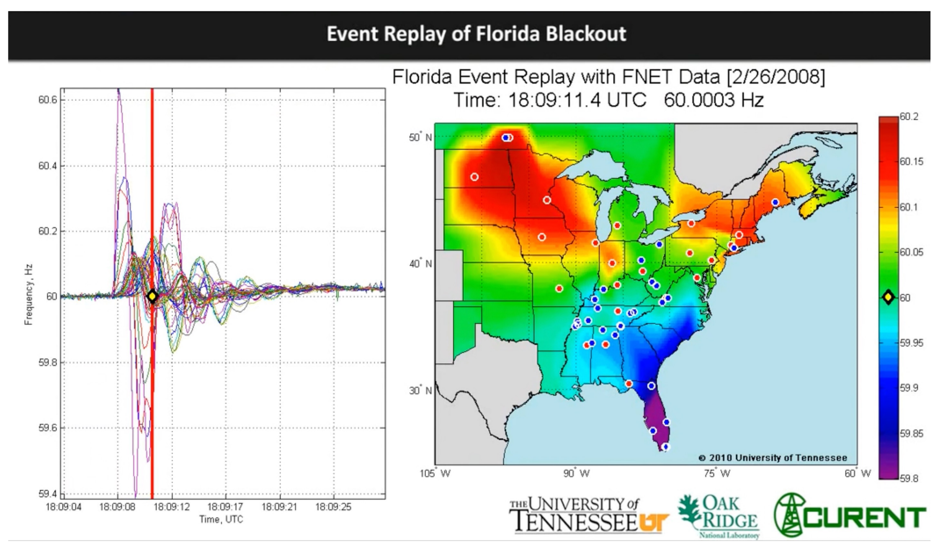
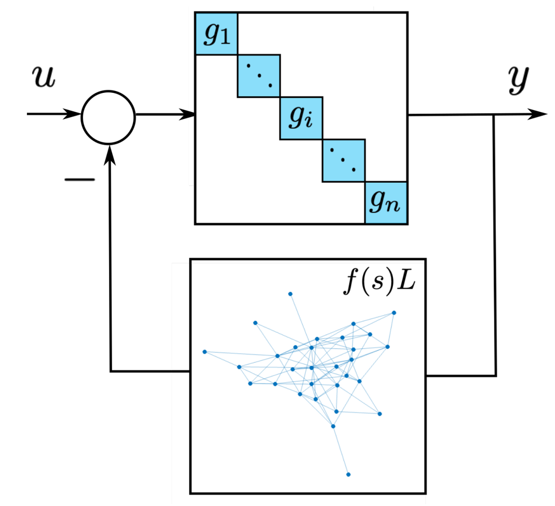
NETWORK COHERENCE

- In large-scale networks, there may exist multiple coherent areas. Within each area, nodes have similar response
- **Goal:**
  - Understand how coherence emerges in network systems
  - Learn coarse/reduced models of network response
- **Prior work[2-4]:**
  - Assumes 1st or 2nd order node dynamics; No theoretical guarantees



PRELIMINARIES[1]: WHY COHERENCE EMERGES?



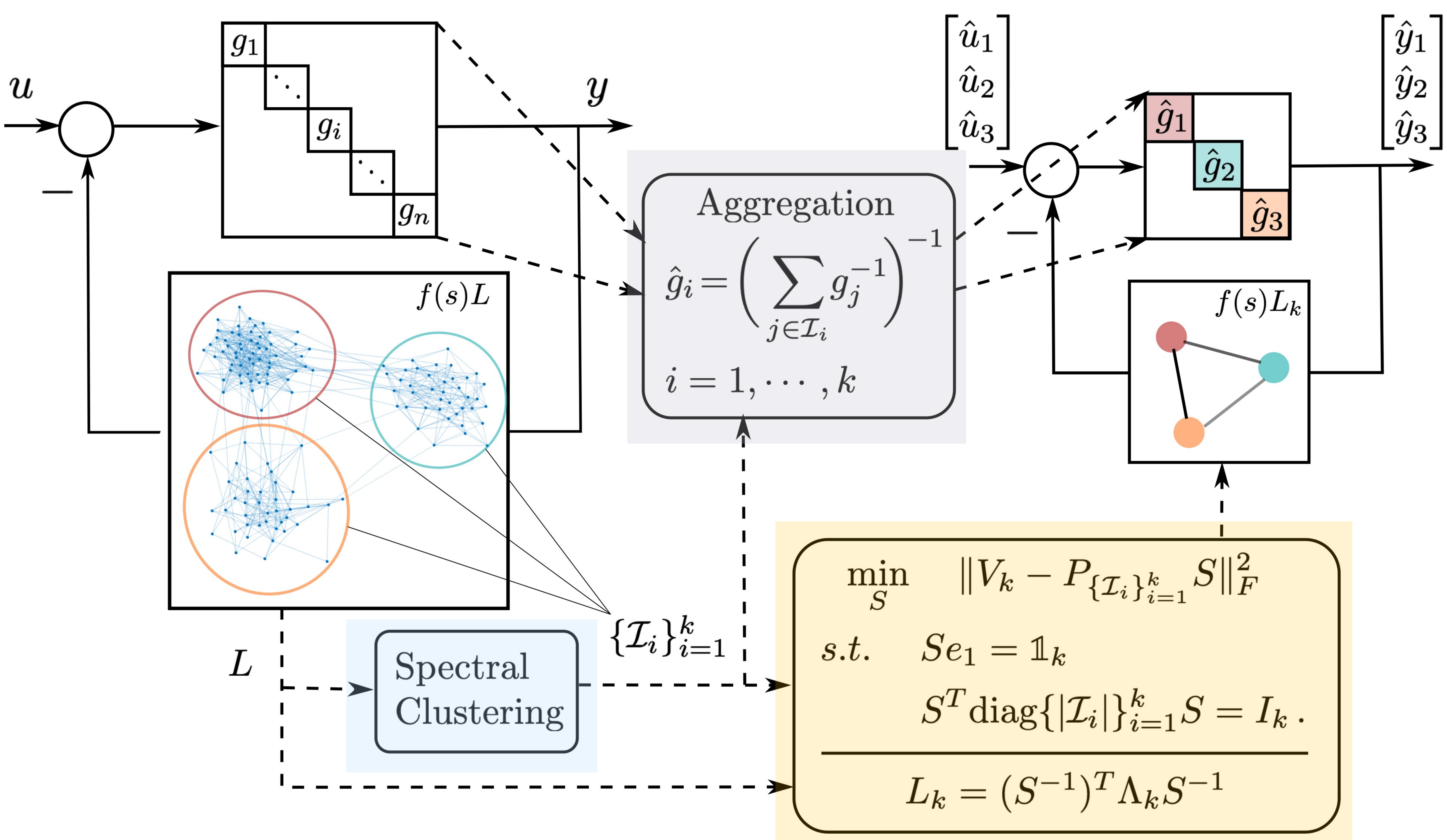
Node dynamics: $g_i(s)$
Real symmetric Laplacian: L
Coupling dynamics: $f(s)$

Network transfer matrix $T(s)$

$$\|T(s_0)-\hat{g}(s_0)\mathbb{1}\mathbb{1}^T\|_2 \sim \mathcal{O}\left(\frac{1}{\lambda_2(L)}\right),\quad \hat{g}(s)=\left(\sum_{i=1}^n g_i^{-1}(s)\right)^{-1}$$

- Coherence emerges when network connectivity increases
- Coherent dynamics is characterized by $\hat{g}(s)$

HOW TO LEARN A COARSE MODEL OF THE COHERENT RESPONSE IN WEAKLY-CONNECTED NETWORKS?



Our Algorithm:

- **Learning coherent areas** by finding tightly-connected subnetworks (**Spectral Clustering**)
- **Aggregate** all nodes in a coherent area into one
- Construct a **reduced network** to model the interaction among the aggregate nodes

Error Analysis:

- **Approximation error** depends on
  - Whether the network has a multi-cluster structure
  - How well one identifies the coherent areas and model the interaction

$s_0\in\mathbb{C}$: "frequency" of inputs of interest

$$\left\|T(s_0)-\hat{T}_k(s_0)\right\|_2 \sim \mathcal{O}\left(\frac{1}{\lambda_{k+1}(L)}\right)+\mathcal{O}\left(\left\|V_k(L)-P_{\{I_i\}_{i=1}^k}S\right\|_2\right)$$

original network — reduced network

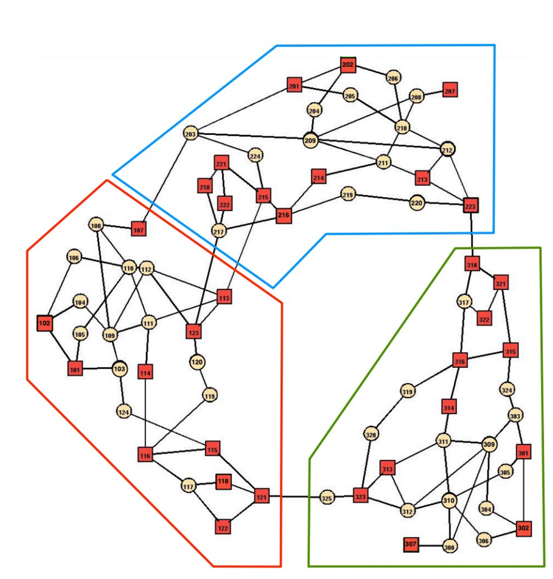
Summary:

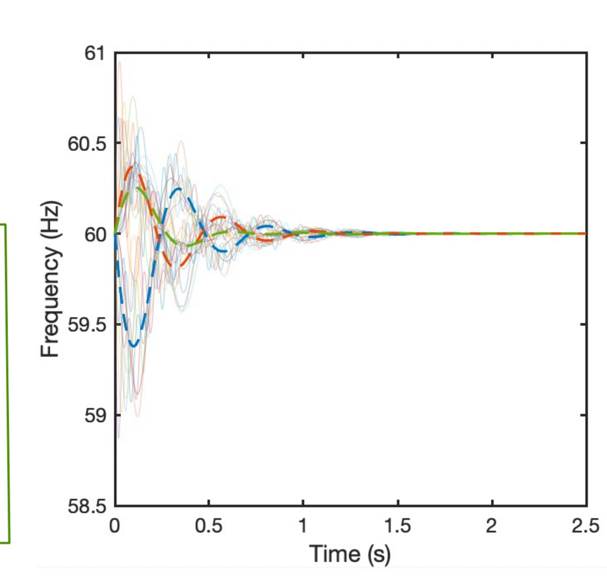
- **Spectral-clustering-**based coherence identification
- Area aggregation for **general** node dynamics
- **Structure-preserving** model reduction
- Error bounds and **theoretical guarantees**

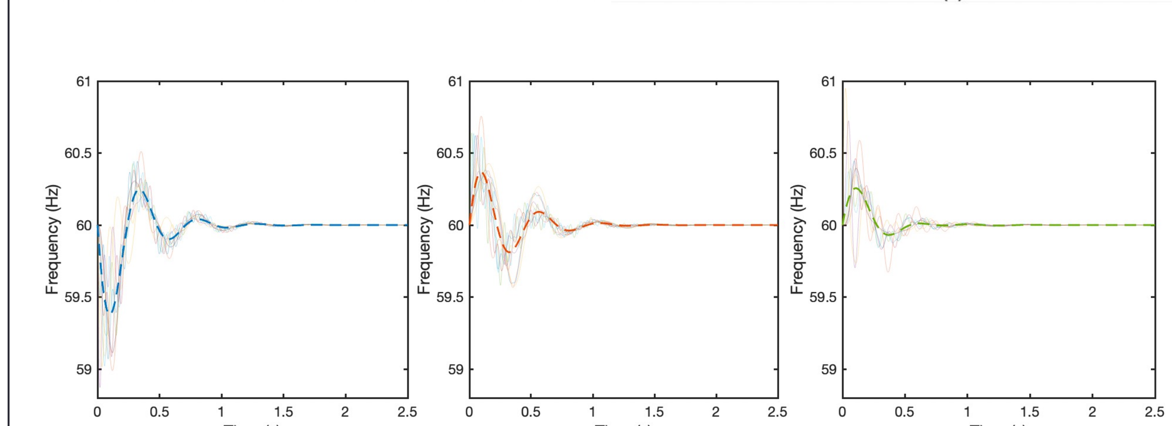
TIME DOMAIN GUARANTEES?

- The network is a random sample from a k-**community stochastic block model** (Assumption 1,2,3,...), and the network size is sufficiently large.
- In **frequency domain**: $\sup_{s\in(-j\eta,+j\eta)}\left\|T(s)-\hat{T}_k(s)\right\|_2 \ll 1.$
- In **time domain**, $T(s)$ and $\hat{T}_k(s)$ has similarresponse when subject to **low-frequency** inputs[1]: $\|y(t)-\hat{y}(t)\|_{\mathcal{L}_\infty} \ll 1.$

NUMERICAL VALIDATION







- The IEEE reliability test system: 1996
- 3 areas, 33 generators in total
- Different rotor angles across each area at initialization
- Solid lines: actual frequency response
  Dashed lines: reduced model

References

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